

## DEPARTMENT OF MATHEMATICS (S.F)



## Vision

To empower women globally competent with human values and ethics acquiring academic and entrepreneurship skills through holistic education.

## Mission

- To create opportunities which will ensure academic excellence in critical thinking, humanistic and scientific inquiry.
- To develop application-oriented courses with the necessary input of values.
- To create a possible environment for innovation, team spirit and entrepreneurial leadership.
- To form young women of competence, commitment and compassion


## PG PROGRAMME EDUCATIONAL OBJECTIVES (PEOs)

| POs | Upon completion of M. Sc. Degree Programme, the graduates will be <br> able to: | Mapping <br> with <br> Mission |
| :--- | :--- | :--- |
| PEO1 | apply scientific and computational technology to solve social and <br> ecological issues and pursue research. | $\mathrm{M} 1, \mathrm{M} 2$ |
| PEO2 | continue to learn and advance their career in industry both in private <br> and public sectors. | M 4 \& M5 |
| PEO2 | develop leadership, teamwork, and professional abilities to become a <br> more cultured and civilized person and to tackle the challenges in <br> serving the country. |  <br> \& 6 |

## PG PROGRAMME OUTCOMES (POs)

| POs | Upon completion of M.Sc. Degree Programme, the graduates <br> will be able to: | Mapping with <br> PEOs |
| :--- | :--- | :--- |
| PO1 | apply their knowledge, analyze complex problems, think <br> independently, formulate and perform quality research. | PEO1 \& PEO2 |
| PO2 | carry out internship programmes and research projects to <br> develop scientific and innovative ideas through effective <br> communication. | PEO1, PEO2 \& PEO3 |
| PO3 | develop a multidisciplinary perspective and contribute to the <br> knowledge capital of the globe. | PEO2 |
| PO4 | develop innovative initiatives to sustain ecofriendly <br> environment | PEO1, PEO2 |
| PO5 | through active career, team work and using managerial skills <br> guide people to the right destination in a smooth and efficient <br> way. | PEO2 |
| PO6 | employ appropriate analysis tools and ICT in a range of learning <br> scenarios, demonstrating the capacity to find, assess, and apply <br> relevant information sources. | PEO1, PEO2 \& PEO3 |
| PO7 | learn independently for lifelong executing professional, social <br> and ethical responsibilities leading to sustainable <br> development. | PEO3 |

## I PG

## Programme Specific Outcomes (PSOs)

| PSO | Upon completion of M.Sc. Degree Programme, the graduates of <br> Mathematics will be able to : | Addressed |
| :--- | :--- | :--- |
| PSO - 1 | Acquire good knowledge and understanding, to solve specific theoretical <br> \& applied problems in different area of mathematics \& statistics | PO1 \& PO2 |
| PSO - $\mathbf{2}$ | Understand, formulate, develop mathematical arguments, logically and <br> use quantitative models to address issues arising in social sciences, <br> business and other context/fields. | PO3 \& PO5 |
| PSO - 3 | Prepare the students who will demonstrate respectful engagement with <br> other's ideas, behaviors, beliefs and apply diverse frames of references <br> to decisions and actions | PO6 |
| PSO - 4 | Pursue scientific research and develop new findings with global <br> impact using latest technologies. | PO4 \& PO7 |
| PSO - 5 | Possess leadership, teamwork and professional skills, enabling them to <br> become cultured and civilized individuals capable of effectively <br> overcoming challenges in both private and public sectors | PO5\& PO7 |

## Teaching Plan

Department
Class
Title of the Course
Semester
Course Code
: Mathematics (SF)
: I M.Sc. Mathematics (SF)
: Core I: Algebraic Structures
: I
: MP231CC1

| Course Code | L | T | $\mathbf{P}$ | Credits | Inst. Hours | Total <br> Hours | Marks |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | CIA | External | Total |
| MP231CC1 | 6 | 1 | - | 4 | 7 | 90 | 25 | 75 | 100 |

## Objectives

1. To introduce the concepts and to develop working knowledge on class equation, solvability of groups.
2. To understand the concepts of finite abelian groups, linear transformations, real quadratic forms

## Course outcomes

| CO | Upon completion of this course, the students will be able to: | PSO <br> addressed | Cognitive <br> level |
| :--- | :--- | :---: | :---: |
| CO-1 | Students will be able to recall basic counting principle, define <br> class equations to solve problems, explain Sylow's theorems and <br> apply the theorem to find number of Sylow subgroups. | PSO - 1 | K1 (R) |
| CO-2 | Define Solvable groups, define direct products, examine the <br> properties of finite abelian groups, define modules | PSO - 2 | K2 (U) |
| CO-3 | Define similar Transformations, define invariant subspace, <br> explore the properties of triangular matrix, to find the index of <br> nilpotence to decompose a space into invariant subspaces, to find <br> invariants of linear transformation, to explore the properties of <br> nilpotent transformation relating nilpotence with invariants. | PSO - 3 | K3 (A) |
| CO - 4 | Define Jordan, canonical form, Jordan blocks, define rational <br> canonical form, define companion matrix of polynomial, find the | PSO - 3 | K3 (A) |


|  | elementary devices of transformation, apply the concepts to find <br> characteristic polynomial of linear transformation. |  |  |
| :--- | :--- | :--- | :--- |
| CO-5 | Define trace, define transpose of a matrix, explain the properties <br> of trace and transpose, to find trace, to find transpose of matrix, <br> to prove Jacobson lemma using the triangular form, define <br> symmetric matrix, skew symmetric matrix, adjoint, to define <br> Hermitian, unitary, normal transformations and to verify whether <br> the transformation in Hermitian, unitary and normal | K5 (E) |  |
|  |  |  |  |

Total Contact hours: 90 (Including lectures, assignments and tests)

| Unit | Module | Topics | Teaching <br> Hours | Cognitive <br> level | Pedagogy | Assessment/ <br> Evaluation |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| I | 1 |  | Another <br> Counting <br> Principle - <br> Definition of <br> conjugate and <br> normalizer, <br> Lemma on <br> conjugacy and <br> normalizer. | 3 | K1 (R) | Introductor <br> y session, <br> Lecture <br> with <br> illustration | Questioning, <br> Recall steps, <br> concept <br> definitions, <br> concept with <br> examples |
|  | 2 | Theorems and <br> Corollary on <br> conjugate <br> class, problems <br> of another <br> counting <br> principle | 4 | K5 (E) | Group <br> Discussion, <br> Lecture <br> with <br> illustration, | Evaluation <br> through short <br> test, concept <br> explanations, <br> solve <br> problems |  |
|  |  |  | Froblem <br> solving | pirst part of <br> Sylow's <br> Theorem- First <br> proof, | 4 | K3 (A) | Lecture <br> with <br> illustration, |
| 3 | Slip Test, <br> concept <br> explanations |  |  |  |  |  |  |


|  |  | Corollary and Lemma on pSylow subgroup. |  |  | Peer tutoring |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4 | Second and <br> Third part of <br> Sylow's <br> Theorem, <br> problems of <br> Sylow's <br> Theorem | 4 | K5 (E) | Lecture with illustration, PPT, Problem solving | Quiz, concept explanations, solve problems |
| II |  |  |  |  |  |  |
|  | 1 | Direct <br> Products- <br> Definition of internal direct product, Lemma and Theorem on internal direct product, problems on direct products | 3 | $\begin{aligned} & \text { K2 (U) } \\ & \text { K5 (E) } \end{aligned}$ | Introductor y session, Lecture with illustration, Problem solving | Recall steps, Questioning, concept definitions, concept with examples, solve problems |
|  | 2 | Finite Abelian GroupsTheorem based on direct | 4 | K3 (A) | Lecture with illustration, Group Discussion | Discussion, Quiz, concept explanations |


|  |  | product of <br> cyclic groups. |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | Modules- <br> Definition and <br> Examples, <br> Definition of <br> direct sum of <br> submodules <br> and cyclic R- <br> module, <br> Theorem and <br> Corollary <br> based on direct <br> sum of <br> submodules. | 4 | K2 (U) | Lecture <br> with <br> illustration | concept <br> definitions, <br> concept with <br> examples |
| 1 | III | Triangular <br> Form- <br> Definition of | 4 | K2 (U) | Lecture <br> with <br> illustration | concept <br> definitions, <br> concept with |
|  |  |  |  |  |  |  |
| 4 | Solvability by <br> Radicals- <br> Definition on <br> solvable, <br> Lemma and <br> Theorem on <br> solvable, <br> problems on <br> solvability by <br> radicals | 4 | K5 (E) | Lecture <br> with <br> illustration, <br> Problem <br> solving | concept <br> definitions, <br> concept with <br> examples, <br> Evaluation <br> through short <br> test, solve <br> problems |  |


|  |  | similar and invariant, Lemma on invariant |  |  |  | examples, Questioning, Discussion |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | Theorems on triangular, problems on triangular form | 3 | K5 (E) | Lecture with illustration, Peer tutoring, problem solving | concept explanations, Evaluation through short test, solve problems |
|  | 3 | Nilpotent <br> Transformation s- <br> Definition of index of nilpotence, invariant and cyclic, <br> Lemma based on nilpotent, invariant | 4 | K3 (A) | Lecture <br> with illustration, Group Discussion | concept definitions, concept explanations, Quiz |
|  | 4 | Theorems based on index of nilpotence, similar, invariants | 4 | K3 (A) | Lecture with illustration, PPT | concept explanations, slip test |
| IV |  |  |  |  |  |  |
|  | 1 | Jordan Form Definition, Lemma and Corollary on | 4 | K2 (U) | Lecture with illustration | concept definitions, concept with |


|  |  | minimal <br> polynomial |  |  |  | examples, <br> Assignment |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Theorems and <br> Corollary <br> based on <br> invariant, <br> minimal <br> polynomial, <br> Jordan block, <br> problems on <br> Jordan Form | 4 | K5 (E) | Lecture <br> with <br> illustration, <br> Peer <br> tutoring, <br> problem <br> solving | concept <br> explanations, <br> Quiz, solve <br> problems |  |
| 3 | Rational <br> Canonical <br> Form- <br> Definition of <br> companion <br> matrix, Lemma <br> on companion <br> matrix | 3 | K3 (A) |  | Lecture <br> with <br> illustration | concept <br> explanations, <br> Evaluation <br> through short |
| 4 | test |  |  |  |  |  |
| Definition of <br> rational <br> canonical form, <br> elementary <br> divisors, <br> characteristic <br> polynomial, <br> Theorems <br> based on <br> elementary | 4 | K2 (U) |  | Lecture <br> with <br> illustration, <br> Group <br> Discussion | concept <br> definitions, <br> concept <br> explanations, |  |


|  |  | divisors, similar |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V |  |  |  |  |  |  |
|  | 1 | Trace and <br> TransposeDefinition, Lemma and Corollary based on trace, nilpotent, transpose | 3 | K2 (U) | Introductor <br> y session, <br> Lecture <br> with <br> illustration | concept explanations, concept definitions, concept with examples |
|  | 2 | Hermitian, <br> Unitary and <br> Normal <br> Transformation <br> s- Definition, <br> Lemma on unitary, <br> Hermitian <br> adjoint, normal | 4 | K3 (A) | Lecture with illustration | concept definitions, concept explanations, slip Test, seminar |
|  | 3 | Theorems based on unitary, Hermitian, normal, problems on Hermitian | 4 | K5 (E) | Lecture with illustration, problem solving | concept explanations, Quiz, solve problems, seminar |


|  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | Real Quadratic <br> Forms- <br> Definition, <br> Lemma and <br> Theorem on <br> congruent | 4 | K3 (A) | Lecture <br> with <br> illustration | concept <br> definitions, <br> Evaluation <br> through short <br> test, seminar |  |

Course Focussing on Employability/ Entrepreneurship/ Skill Development: Skill Development
Activities (Em/En/SD): Group discussion
Course Focussing on Cross Cutting Issues (Professional Ethics/ Human Values/Environment Sustainability/ Gender Equity): Nil

Activities related to Cross Cutting Issues: Nil
Assignment: Jordan Form
Seminar Topic: Hermitian, Unitary and Normal Transformations

## Sample questions

## Part A

1. If $a \epsilon G$ then the normalizer of $a$ in G , is the set $\mathrm{N}(\mathrm{a})=$ $\qquad$ .
2. An R -module M is said to be $\qquad$ if there is an element $m_{0} \in M$ such that every $m \in M$ is of the form $m=r m_{0}$ where $r \in R$.
(a) Finitely generated
(b) cyclic
(c) direct sum
(d) unital
3. State True or False: The subspace W of V is invariant under $T \in A(V)$ if $\mathrm{WT} \subset \mathrm{W}$.
4. If $\operatorname{dim}_{F}(V)=n$ then the $\qquad$ of T , is the product of its elementary divisors.
(a) characteristic polynomial
(b) companion matrix
(c) rational canonical form
(d) similar
5. Two real symmetric matrices A and B are $\qquad$ if there is a nonsingular real matrix T such that $B=$ TAT'.

## Part B

1. Prove that $N(a)$ is a subgroup of $G$.
2. Prove that G is solvable if and only if $G^{k}=(e)$ for some integer k .
3. If M of dimension m is cyclic with respect to T then prove that the dimension of $M T^{k}$ is $\mathrm{m}-\mathrm{k}$ for all $k \leq$ $m$.
4. Suppose that $V=V_{1} \oplus V_{2}$ where $V_{1}$ and $V_{2}$ are subspaces of V invariant under T . Let $T_{1}$ and $T_{2}$ be the linear transformations induced by T on $V_{1}$ and $V_{2}$ respectively. If the minimal polynomial of $T_{1}$ over F is $p_{1}(x)$ while that of $T_{2}$ is $p_{2}(x)$ then prove that the minimal polnomial for T over F is the least common multiple of $p_{1}(x)$ and $p_{2}(x)$.
5. If $T \in A(V)$ then $\operatorname{tr} \mathrm{T}$ is the sum of the characteristic roots of T .

## Part C

1. Prove that the number of p -Sylow subgroups in G , for a given prime, is of the form $1+\mathrm{kp}$.
2. Prove that every finite abelian group is the direct product of cyclic groups.
3. If $\mathrm{T} \in A(V)$ has all its characteristic roots in F then prove that there is a basis of V in which the matrix of T is triangular.
4. Prove that the elements S and T in $A_{F}(V)$ are similar in $A_{F}(V)$ if and only if they have the same elementary divisors.
5. Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V .


Head of the Department: Dr.S.Kavitha


Course Instructor: Dr.C.Jenila

## Teaching Plan

| Department | $: \quad$ Mathematics |
| :--- | :--- |
| Class | $: \quad$ I M.Sc. Mathematics |
| Title of the Course | $:$ |
| Semere II $:$ Real Analysis I |  |
| Course Code | $: \quad$ I |
|  | $: \quad$ MP231CC2 |


| Course <br> Code | L | T | P | Credits | Inst. <br> Hours | Total <br> Hours | Marks |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | CIA | External | Total |
| MP231CC2 | 5 | 2 | - | 4 | 7 | 105 | 25 | 75 | 100 |

## Learning Objectives:

1. To work comfortably with functions of bounded variation, Riemann- Stieltjes Integration, convergence of infinite series, infinite product and uniform convergence.
2. To relate its interplay between various limiting operations

## Course outcomes

| CO | Upon completion of this course, the <br> students will be able to: | PSO addressed | Cognitive level |
| :--- | :--- | :--- | :--- |
| CO - | Analyze and evaluate functions of <br> bounded variation and rectifiable <br> Curves. | PSO -1 | K4, K5 |
| CO - 2 | Describe the concept of Riemann- <br> Stieltjes integrals and its properties. | PSO -2 | K1, K2 |
| CO -3 | Demonstrate the concept of step <br> function, upper function, Lebesgue <br> function and their integrals. | PSO -2 | K3 |
| CO - 4 | Construct various mathematical proofs <br> using the properties of Lebesgue <br> integrals and establish the Levi <br> monotone convergence theorem. | PSO -4 | K3, K5 |


| CO - 5 | Formulate the concept and properties of <br> inner products, norms and measurable <br> functions. | PSO - 2 | K2, K3 |
| :--- | :--- | :--- | :--- |

Total contact hours: 90 (Including instruction hours, assignments and tests)

| Unit | Module | Topic | Teaching Hours | Cognitive level | Pedagogy | Assessment/ Evaluation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | Functions of Bounded Variation, Infinite Series |  |  |  |  |  |
|  | 1. | Definition of monotonic function connected and disconnected functions compact sets and examples | 2 | K1, K2 | Recall the basic definitions | Questioning |
|  | 2. | Properties of monotonic functions, Functions of bounded variation, Definition - Partition, Bounded variation, Examples of continuous functions that are not of bounded variation, Illustration on boundedness of $f^{\prime}$ is not necessary for $f$ to be of bounded variation | 4 | K4, K5 | Lecture with illustration | Summarize the concepts |
|  | 3. | Total variation - Definition, Behaviour of functions of bounded variation, Example illustrating reciprocal of functions of total variation need not be of total variation, Additive property of total variation | 3 | K2, K5 | Illustrative Method | Questioning |
|  | 4. | Total variation on $[\mathrm{a}, \mathrm{x}]$ as a function of the right end point x, Functions of bounded variation expressed as the difference of increasing functions Characterisation of functions of bounded variation, Continuous | 4 | K2, K5 | Lecture | Question and answer |


|  |  | functions of bounded variation |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5. | Absolute and Conditional convergence, Definition Absolutely convergent series, Example illustrating convergence does not imply absolute convergence, Dirichlet's test and Abel's test | 3 | K2, K4 | Illustrative method and Discussion | Slip test |
|  | 6. | Rearrangement of series, Riemann's theorem on conditional convergent series | 3 | K4 | Lecture | Class test |
| II | The Riemann - Stieltjes integral |  |  |  |  |  |
|  | 1. | The Riemann - Stieltjes integral - Introduction, Basics of calculus, Notation, Definition refinement of partition, norm of a partition, The definition of The Riemann Stieltjes integral, integrand, integrator, Riemann integral | 3 | K1 | Brainstorming | Questioning |
|  | 2. | Linear properties of Riemann - Stieltjes integral, Integration by parts, Connection between integrand and the integrator in a Riemann - Stieltjes integral | 3 | K2 | Discussion and Lecture | Slip test |
|  | 3. | Change of variable in a Riemann - Stieltjes integral, Reduction to a Riemann integral, Step functions as integrators, Example showing that the existence of integral can also be affected by a change | 4 | K2 | Flipped <br> Classroom | Q \& A |
|  | 4. | Reduction of a Riemann Stieltjes integral to a finite sum, Definition - Step function, Euler's Summation formula, Monotonically increasing integrators, upper and lower integrals, Definition - upper | 4 | K2 | Lecture | Quiz method |


|  |  | and lower Stieltjes sums of f with respect to $\alpha$ for the partition P, Theorem illustrating for increasing $\alpha$, refinement of partition increases the lower sums and decreases the upper sums |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 5. | Definition - Upper and lower Stieltjes integral, upper and lower Riemann sums, Examples, Additive and linearity properties of upper and lower integrals, Riemann's condition, Comparison theorems | 4 | K2 | Illustration method | MCQ |
| III | The Riemann - Stieltjes integral |  |  |  |  |  |
|  | 1. | Integrators of bounded variation, Sufficient conditions for existence of Riemann - Stieltjes integrals | 3 | K2 | Lecture | Short test |
|  | 2. | Necessary conditions for existence of Riemann Stieltjes integrals, Theorem illustrating common discontinuities from the right or from the left, Mean - value theorems for Riemann - Stieltjes integrals - first mean value theorem, second mean - value theorem, the integral as a function of the interval and its properties | 4 | K3, K4 | Lecture | Problemsolving |
|  | 3. | Second fundamental theorem of fundamental calculus, Change of variable in a Riemann integral, Second Mean - Value theorem for Riemann integrals | 4 | K3, K4 | Lecture | Short test |
|  | 4. | Riemann - Stieltjes integrals depending on a parameter, Differentiation under the integral sign | 3 | K3, K5 | Lecture | Questioning |


|  | 5. | Interchanging the order of integration, Lebesgue's criterion for existence of Riemann integrals, Definition - measure zero, examples, Definition oscillation of $f$ at $x$, oscillation of f on T , Lebesgue's criterion for Riemann integrability | 4 | K4 | Interactive method | Slip Test |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IV | Infinite Series and Infinite Products, Power Series |  |  |  |  |  |
|  | 1. | Double sequences, Definition - Double sequence, convergence of double sequence, Example, Definition - Uniform convergence, Double series, Double series and its convergence, Rearrangement theorem for double series, Definition Rearrangement of double sequence | 3 | K1 \& K2 | Brainstorming PPT | Quiz |
|  | 2. | A sufficient condition for equality of iterated series, Multiplication of series, Definition - Product of two series, Conditionally convergent series, Cauchy product, Merten's Theorem, Dirichlet product | 5 | K3 | Lecture | True/ False |
|  | 3. | Cesaro Summability, Infinite products, Definition - infinite products, Cauchy condition for products | 4 | K2 | Lecture | Concept Explanation |
|  | 4. | Power series, Definition Power series, Multiplication of power series, Definition - Taylor's series | 3 | K3, K4 | Lecture with chalk and talk | Slip Test |
|  | 5. | Abel's limit theorem, Tauber's theorem | 3 | K2, K4 | Lecture Discussion | Q \& A |
| V |  |  |  | unctions |  |  |
|  | 1. | Sequences of function Pointwise convergence of sequence of function, Examples of sequences of real valued functions | 3 | K2 | Introductory Session | Explain |


|  | 2. | Uniform convergence and <br> continuity, Cauchy <br> condition for uniform <br> convergence | 4 | K2, K4 | Lecture with <br> illustration | Concept <br> explanations |
| :---: | :---: | :--- | :--- | :--- | :--- | :--- |
|  | 3. | Uniform convergence of <br> infinite series of functions, <br> Riemann - Stieltjes <br> integration, Non-uniform <br> convergence and term-by- <br> term integration | 2 | K3, K4 | Seminar <br> Presentation | Questioning |
|  | 4. | Uniform convergence and <br> differentiation, Sufficient <br> condition for uniform <br> convergence of a series, <br> Mean convergence | 4 | K2 | Seminar <br> Presentation | Recall steps |

Course Focussing on Employability/ Entrepreneurship/ Skill Development: Skill Development
Activities (Em/En/SD): Problem-solving, Seminar Presentation, Group Discussion
Course Focussing on Cross Cutting Issues (Professional Ethics/ Human Values/Environment Sustainability/ Gender Equity): -Nil

Activities related to Cross Cutting Issues: -Nil
Assignment: Solving Exercise Problems
Seminar Topic: Double Sequence, Double Series, Rearrangement theorem for Double series, Sequence of functions- Pointwise, Uniform convergence

## Sample questions

## Part A

1. The function $f$ is of bounded variation on $[a, b]$ if $f$ is $\qquad$ on [a,b].
2. Riemann integral is a special case of Riemann Stieltjes integral when $\alpha(x)=$ $\qquad$ .
3. Say true or false: Assume that $\alpha \nearrow$ on $[\mathrm{a}, \mathrm{b}]$. If $f \in R(\alpha)$ on $[\mathrm{a}, \mathrm{b}]$, then $f^{2} \in R(\alpha)$ on $[\mathrm{a}, \mathrm{b}]$.
4.Define a double sequence.
5.A sequence of functions $\left\{f_{n}\right\}$ is said to be $\qquad$ on T if $\left\{f_{n}\right\}$ is pointwise convergent and uniformly bounded on $T$.

## Part B

1. Prove that if $f$ is monotonic on $[a, b]$, then the set of discontinuities of $f$ is countable.
2. Prove that the Riemann Stieltjes integral operates in a linear fashion on the integrator.
3.Prove that if f is continuous on $[\mathrm{a}, \mathrm{b}]$ and if $\alpha$ is of bounded variation on [a, b$]$, then $f \in R(\alpha)$ on $[\mathrm{a}, \mathrm{b}]$.
4.State and prove Cauchy condition for products.
5.State and prove Dirichlet's test for uniform convergence.

## Part C

1. State and prove Riemann's theorem on conditionally convergent series.
2. State and prove the formula for Integration by parts.
3.Assume that $\alpha$ is of bounded variation on [a,b]. Let $\mathrm{V}(\mathrm{x})$ denote the total variation of $\alpha$ on $[\mathrm{a}, \mathrm{x}]$ if $a<\mathrm{x} \leq b$, and let $\mathrm{V}(\mathrm{a})=0$. Let f be defined and bounded on [a,b]. If $\in R(\alpha)$ on [a, b$]$, then prove that $\in R(V)$ on $[\mathrm{a}, \mathrm{b}]$.
3. State and prove Merten's theorem.
4. Assume that each term of $\left\{f_{n}\right\}$ is a real-valued function having a finite derivative at each point of an open interval (a,b). Assume that for at least one point $x_{0}$ in $(\mathrm{a}, \mathrm{b})$ the sequence $f_{n}\left(x_{0}\right)$ converges. Assume further that there exists a function g such that $f_{n}{ }^{\prime} \rightarrow g$ uniformly on $(\mathrm{a}, \mathrm{b})$. Then prove that
a) There exists a function f such that $f_{n} \rightarrow f$ uniformly on (a,b).
b) For each x in $(\mathrm{a}, \mathrm{b})$ the derivative $f^{\prime}(x)$ exists and equals $\mathrm{g}(\mathrm{x})$.


Head of the Department

## Dr. S.Kavitha

# J. Anne Mary Leema 

Course Instructor
Mrs.J.Anne Mary Leema

# Ordinary Differential Equations 

Department
Class
Title of the Course
Semester
Subject code
: Mathematics(SF)
: I M.Sc
: Ordinary Differential Equations
: I
: MP231CC3

| Course <br> Code | L | T | P |  | S |  | Credits |  | Inst. <br> Hours |  | Total <br> Hours | Marks |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | CIA | External |  |  | Total |
| MP231CC3 | 5 | 1 | - |  |  |  |  |  | 4 |  |  | 6 |  | $\begin{aligned} & 9 \\ & 0 \end{aligned}$ | 25 | 75 | 100 |

## Learning Objectives:

1. To develop strong background on finding solutions to linear differential equations with constant variable coefficients and also with singular points.
2. To study existence and uniqueness of the solutions of first order differential equation.
3. To study mathematical methods for solving differential equations
4. Solve dynamical problems of practical interest.

## Course outcomes

| CO | Upon completion of this course, <br> the students will be able to: | PSO addressed | Cognitive level |
| :---: | :--- | :---: | :---: |
| $\mathrm{CO}-1$ | Establish the qualitative behaviour <br> of solutions of systems of <br> differentialequations. | PSO -1 | K3 |
| $\mathrm{CO}-2$ | Recognize the physical phenomena <br> modelled by differential equations <br> anddynamical systems. | PSO -2 | K1 |
| $\mathrm{CO}-3$ | Analyze solutions using appropriate <br> methods and give examples. | PSO -3 | K4 |


| $\mathrm{CO}-4$ | Formulate Green's function for <br> boundary value problems. | PSO -3 | K5 |
| :--- | :--- | :---: | :---: |
| $\mathrm{CO}-5$ | Understand and use the various <br> theoretical ideas and results that <br> underlie themathematics in course. | PSO -3 | K2 |

K1- Remember K2 - Understand K3-Apply K4- Analyze K5-Evaluate

Total contact hours: 75 (Including lectures, assignments and tests)

| Unit | Mod ule | Topic | Teaching Hours | Cogni tive level | Pedagogy | Assessment/ <br> Evaluation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | Linear equations with constant coefficients: |  |  |  |  |  |
|  | 1 | Second order homogeneous Equations Introduction | 4 | K1(R) | Lecture using Chalk and talk ,Introductory session. | Simple definitions, Recall steps, Concept definitions |
|  | 2 | Initial value problems | 4 | K2(U) | Group Discussion, Lecture using videos, Problem solving | Evaluation through short test, Concept explanations |
|  | 3 | Linear dependence and independence | 4 | K2(U) | Peer tutoring, Lecture using videos | Suggest formulae, Solve problems, Explain. |
|  | 4 | Wronskian and a formula for Wronskian Nonhomogeneous equation of order two. | 4 | $\begin{aligned} & \text { K3(A } \\ & \text { p) } \end{aligned}$ | Lecture using Chalk and talk, Problem solving | Evaluation through short test, Seminar. |
| II | Linear equations with constant coefficients: |  |  |  |  |  |
|  | 1 | Homogeneous and non-homogeneous equation of order $n$ | 4 | K2(U) | Lecture using Chalk and talk , Group Discussion, Mind mapping, Problem solving. | Problem-solving questions, <br> Evaluation through short test. |



| V | Existence and uniqueness of solutions to first order equations: |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | Equation with variable separated | 3 | K2(U) | Lecture using Chalk and talk | Concept explanations |
|  | 2 | Exact equation method of successive approximations | 3 | $\begin{aligned} & \mathrm{K} 2(\mathrm{~A} \\ & \mathrm{p}) \end{aligned}$ | Group Discussion, Problem solving | Problem-solving questions |
|  | 3 | The Lipschitz condition | 4 | K2(U) | Problem solving Method, Peer tutoring | Problem-solving questions |
|  | 4 | Convergence of the successive approximations and the existence theorem. | 5 | $\begin{aligned} & \text { K3(A } \\ & \text { p) } \end{aligned}$ | Group Discussion, Lecture using Chalk and talk | Problem-solving questions |

Course Focussing on Employability/ Entrepreneurship/ Skill Development : Skill Development
Activities (Em/En/SD): Solving the Problems, Group discussion, Seminar, Online Assignment
Course Focussing on Cross Cutting Issues (Professional Ethics/ Human Values/Environment Sustainability/
Gender Equity): Nil
Activities related to Cross Cutting Issues: Nil
Assignment : Wronskian and linear dependence and reduction of the order of a homogeneous equation
Seminar Topic: Linear equation with regular singular points.

## Sample questions

## Part A

1. The characteristic polynomial $p(r)$ of the equation $L(y)=y^{\prime \prime}+a_{1} y^{\prime}+a_{2} y=0$ is $\qquad$ .
a) $a_{1} r^{2}+a_{2} r+a_{3}=p(r)$
b) $a_{1} r^{3}+a_{2} r^{2}+a_{3} r=p(r)$
c) $r^{2}+a_{1} r+a_{2}=p(r)$
2. For the $n$-th order equations $\|\phi(x)\|=$ $\qquad$ .
a) $\left[|\phi(x)|^{2}+\cdots+\left|\phi^{(n-1)}(x)\right|^{2}\right]^{1 / 2}$
b) $\left[|\phi(x)|^{2}+\cdots+\left|\phi^{(n-1)}(x)\right|^{2}\right]^{1 / 3}$
c) $\left[|\phi(x)|^{2}+\cdots+\left|\phi^{(n-1)}(x)\right|^{2}\right]^{1 / 4}$
3. If $\phi_{1}, \phi_{2}, \ldots, \phi_{n}$ are $n$ solutions of $L(y)=0$ on an interval $I$, they are linearly independent there if , and only if,
a) $\mathrm{W}\left(\phi_{1}, \Phi_{2}, \ldots, \phi_{n}\right)=0$
b) $\mathrm{W}\left(\Phi_{1}, \Phi_{2}, \ldots, \Phi_{n}\right)$
$\left.\phi_{n}\right) \neq 0$
c) $\mathrm{W}\left(\phi_{1}, \phi_{2}, \ldots, \phi_{n-1}\right)=0$
4. The Legendre equation is $\qquad$ .
a) $\mathrm{L}(\mathrm{y})=\left(1-x^{2}\right) "-2 x y^{\prime}+\alpha(\alpha+1) y=0$
b) $\mathrm{L}(\mathrm{y})=\left(1+x^{2}\right)^{\prime \prime}-2 x y^{\prime}+\alpha(\alpha+1) y=0$
c) $\mathrm{L}(\mathrm{y})=$
$\left(1-x^{2}\right) "+2 x y^{\prime}+\alpha(\alpha+1) y=0$
5. The Bessel's equation is $\qquad$ .
a) $x^{2} y^{\prime \prime}-x y^{\prime}+\left(x^{2}-\alpha^{2}\right) y=0$
b) $x^{2} y^{\prime \prime}-x y^{\prime}-\left(x^{2}-\alpha^{2}\right) y=0$
c) $x^{2} y^{\prime \prime}+x y^{\prime}+\left(x^{2}-\alpha^{2}\right) y=0$

## Part B

1. Prove that the infinite series involved in the definition of $\mathrm{K}_{\mathrm{n}}$ converges for $|x|<\infty$.
2. Solve $\cos x \cos ^{2} y d x-\sin x \sin 2 y d y=0$
3. Let $\alpha, \beta$ be any two constants, and let $x_{0}$ be any real number. On any interval I containing $x_{0}$ there exists at most one solution $\phi$ of the initial value problem $\mathrm{L}(\mathrm{y})=0, \mathrm{y}\left(x_{0}\right)=\alpha, \mathrm{y}^{\prime}\left(x_{0}\right)=\beta$.
4. Compute three linearly independent solutions for the equation $y^{\prime \prime \prime}-4 y^{\prime}=0$.
5. One solution of $x^{2} y^{\prime \prime}-2 y=0$ on $0<x<\infty$ is $\phi_{1}(x)=x^{2}$. Find all solutions of $x^{2} y^{\prime \prime}-2 y=2 x-1$ on $0<x<\infty$.

## Part C

1. If $\phi_{1}, \Phi_{2}, \ldots, \phi_{n}$ are n solutions of $\mathrm{L}(\mathrm{y})=0$ on an interval I , they are linearly independent there if , and only if, $W\left(\phi_{1}, \Phi_{2}, \ldots, \Phi_{n}\right) \neq 0$ for all $x$ in .
2. Second order equations with regular singular points - the general case.
3. Let $\mathrm{M}, \mathrm{N}$ be two real-valued functions which have continuous first partial derivatives on some rectangle $\mathrm{R}:\left|x-x_{0}\right| \leq \mathrm{a}, \quad\left|y-y_{0}\right| \leq \mathrm{b}$. then the equation $\mathrm{M}(x, y)+\mathrm{N}(x, y) y^{\prime}=0$ is exact in R if, and only if, $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ in R .
4. Let $\phi$ be any solution of $\mathrm{L}(\mathrm{y})=\mathrm{y}^{\prime \prime}+\mathrm{a}_{1} y^{\prime}+\mathrm{a}_{2} \mathrm{y}=0$ on an interval I containing a point $x_{0}$. Then for all $x$ in I $\left\|\phi\left(x_{0}\right)\right\| e^{-k\left|x-x_{0}\right|} \leq\|\phi(x)\| \leq\left\|\phi\left(x_{0}\right)\right\| e^{k\left|x-x_{0}\right|}$ where $\left[|\phi(x)|^{2}+\left|\phi^{\prime}(x)\right|^{2}\right]^{1 / 2}, \mathrm{k}=1+\left|\mathrm{a}_{1}\right|+\left|\mathrm{a}_{2}\right|$.
5. Consider the equation with constant coefficients $\mathrm{L}(\mathrm{y})=\mathrm{P}(x) e^{a x}$ where P is the polynomial given by $\mathrm{P}(x)$ $=b_{0} x^{m}+b_{1} x^{m-1}+\ldots+b_{m},\left(b_{0} \neq 0\right)$. Suppose $a$ is the root of the characteristic polynomial $p$ of $L$ of multiplicity j . then there is a unique solution $\psi(x)=x^{j}\left(c_{0} x^{m}+c_{1} x^{m-1}+\cdots+c_{m}\right) e^{a x}$, where $c_{0}, c_{1}, \ldots, c_{m}$ are constants determined by the annihilator method.


Head of the Department: Dr.S.Kavitha


Course Instructor: Ms.P.C.Priyanka Nair

# Teaching Plan 

Department
Class
Title of the Course : Elective -I Graph Theory and Application
Semester Course Code
: I M.Sc Mathematics
: Mathematics
: I
: MP231DE1

| Course Code | L | T | P | Credits | Inst. Hours | Total <br> Hours | Marks |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MP231DE1 |  |  |  |  | CIA | External | Total |  |  |

## Objectives

1. To help students to understand various parameters of Graph Theory with applicants.
2. To stimulate the analytical mind of the students, enable them to acquire sufficient knowledge and skill in the subject that will make them component in various areas of Mathematics.
Course outcomes

| CO | Upon completion of this course, the students will be able to: | PSO addressed | Cognitive level |
| :---: | :---: | :---: | :---: |
| CO-1 | Recall the basic concepts of graph theory and know its various parameters. | PSO-1 | K1(R) |
| CO-2 | Understand the many results derived on the basis of known parameters. | PSO-2 | K2(U) |
| CO-3 | Apply the concepts to evaluate parameters for the family of graphs. | PSO-4 | K3(A)\&K4(An) |
| CO-4 | Analyze the steps of various theorems and know its applications. | PSO-3 | K1(R)\&K4(An) |
| CO-5 | Create a graphical model for the real world problem using the relevant ideas. | PSO-4 | K6(C) |

Total Contact hours: 60 (Including lectures, assignments and tests)

| Unit | Module | Topic | Teaching Hours | $\begin{array}{\|c} \hline \text { Cogniti } \\ \text { ve } \\ \text { level } \\ \hline \end{array}$ | Pedagogy | Assessmen t/ Evaluation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I |  |  |  |  |  |  |
|  | 1. | Definition of tree \& examples, theorems.Cut edges definitions, examples \& Theorems. | 3 | K2(U) | Lecture with Illustration | Evaluation through test |
|  | 2. | Definition of bond ,examples, theorems, Definition of cotree \& examples, theorems. | 2 | K1(R) | Lecture with Illustration | Recall concept definitiona nd steps |
|  | 3. | Cut vertices: Definition \& Examples, Theorems, Corollary | 2 | K3(Ap) | Lecture with Examples | Suggest concept with examples, |
|  | 4. | Definition of k-connected, k-edge connected \& examples,theorems | 2 | K4(An) | Discussion with Illustration | Differentia te between various ideas |
|  | 5 | Blocks : Definitions \& examples, theorems, corollary. Contruction of reliable communication network | 2 | K2(U) | Lecture with Illustration | Evaluation through short test |
| II |  |  |  |  |  |  |
|  | 1 |  <br> Examples, <br> Theorems, | 2 | K6(C) | Lecture with PPT | Check knowledge in Tour |
|  | 2 | Hamilton cycles: Definition \& Examples, Theorems, lemma, corollary | 4 | K3(A) | Lecture with illustration | Draw different cycles |
|  | 3 | The Chinese postman <br> problem, Fleury's <br> algorithm,  <br> theorems, problems  | 5 | K2(U) | Lecture using videos | Evaluation through Assessmen t Test |
| III |  |  |  |  |  |  |


|  | 1 | Matching: Defition $\&$ <br> examples, M-saturated,  <br> Maximum matching, M -  <br> alternating path, M - <br> augmenting  path; <br> definitions  $\&$ <br> examples,theorems   | 4 | K2(U) | Lecture with PPT Illustration | Evaluation through Quiz |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | k-edge coloring, k-edge <br> colourable, k-edge <br> chromatic :Definitions, <br> examples, <br> lemma, <br> theorems, <br> number Edge chromatic | 3 | K3(A) | Lecture with Illustration | Suggest concept with examples |
|  | 3 | Bipartite graph: Definition \& example, Vizing's theorem | 2 | K6(C) | Lecture with examples | Check knowledge through Assignmen t |
| IV |  |  |  |  |  |  |
|  | 1 | Independent set, maximum independent set : Definitions, examples \& theorems, corollary | 2 | K2(U) | Lecture withPPT Illustration | Evaluation throughMC Q, Short test |
|  | 2 | Edge covering, edge independence number: Definitions \&examples theorems | 2 | K2(U) | Lecture and group discussion | Evaluation through Quiz |
|  | 3 | k-vertex colouring,k-vertex colourable: Definitions, examples, theorems, corollary | 2 | K4(An) | Lecture with Illustration | Draw different graph |
|  | 4 | k-chromatic graph, kcritical subgraph,clique: Definitions,examples,theor ems, corollary | 3 | K2(U) | Lecture with Illustration | Evauation through Test |
|  | 5 | Brook's theorem <br> Subdivision: definition, Hajo's conjecture, theorem | 2 | K3(A) | Lecture | Explain concept with examples |
| V |  |  |  |  |  |  |
|  | 1 | Definition: embeddable, planar embedding,plane graph,theorems,examples | 3 | K4(An) | Lecture withPPT Illustration | Explain concept with examples |
|  | 2 | Euler's formula: <br> Theorems,corollary | 3 | K5(E) | Lecture with Illustration | Evaluation through Assessmen ttest |


|  | 3 | Kuratowski's theorem, Five <br> colour theorem | 2 | K3(A) | Lecture with <br> PPT <br> Illustration | Evaluation <br> through <br> Sliptest |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 4 | Four colour conjecture | 2 | K3(A) | Lecture with <br> PPT | Explain <br> concept <br> with <br> examples |

Course Focussing on Employability/ Entrepreneurship/ Skill Development : Employability
Activities (Em/ En/SD): group discussion,seminar,Assignment, draw a graph
Course Focussing onCross Cutting Issues(Professional Ethics/ Human Values/Environment Sustainability/Gender Equity): Nil

Activities related to Cross Cutting Issues :Nil
Assignment : Planar Graphs
Seminar Topic: Vertex colouring
Sample questions

## Part A

1. A connected acyclic graph is $\qquad$
2. A Chinese postman problem is finding a $\qquad$
3. G is k-edge chromatic if $\chi^{\prime}(G)=-----$.
4. A subset S of V is called an $\qquad$ if no two vertices of $S$ are adjacent in $G$ i) dependent set ii) independent set
5. Every planar graph is $\qquad$

## Part B

1. Let T be a spanning tree of a connected graph G and let e be any edge of T . Then
i) the cotree $: T$ contains no bond of $G$
ii) T+e contains a unique bond of G
2. If G is a Hamiltonian then for every nonempty proper subset S of $\mathrm{V} \omega(G-S) \leq|S|$
3. If G is bipartite then $\chi^{\prime}=\Delta$.
4. A set $\mathrm{S} \subset V$ is an independent set of G if and only if $\mathrm{V} \backslash \mathrm{S}$ is a covering of G .
5. A graph G is embeddable in the plane if and only if it is embeddable on the sphere.

## Part C

1. Show that $\mathrm{k} \leq k^{\prime} \leq \delta$
2. If G is a simple graph with $\mathrm{V} \geq 3$ and $\delta \geq \frac{\vee}{2}$ then G is a Hamiltonian.
3. A matching M in G is a maximum matching if and only if G contains no $\mathrm{M}-$ augmenting path.
4. Let G be a k - critical graph with 2 -vertex cut $\{\mathrm{u}, \mathrm{v}\}$. Then i) $\mathrm{G}=G_{1} \cup G_{2}$, where $G_{1}$ is a $\{\mathrm{u}, \mathrm{v}\}$ component of type I (i=1,2), and ii) both $G_{1}+u v$ and $G_{2} . u v$ are k- critical.
5. If G is connected plane graph ,then $\vee-\epsilon+\varphi=2$


Head of the Department
Dr. S. Kavitha


Course Instructor
Dr.J.Nesa Golden Flower

## Teaching Plan

Department : Mathematics S.F
Class : I M.Sc Mathematics
Title of the Course : Elective II : Discrete Mathematics
Semester : I
Course Code : MP231GE1

| Course Code | L | T | P | Credits | Inst. Hours | Total <br> Hours | Marks |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4 | 1 |  |  | CIA | External | Total |  |  |
|  | 4 | 1 |  | 5 | 5 | 75 | 25 | 75 | 100 |

## Objectives

1. To learn the concepts of Permutations, Combinations, Boolean Algebra and Lattices
2. To motivate the students to solve practical problems using Discrete Mathematics

Course Outcomes:

| On the successful completion of the course, student will be able to: | PSO <br> addressed | Cognitive |  |
| :---: | :--- | :--- | :--- |
| CO1 | Remember and interpret the basic concepts in permutations and <br> combinations and distinguish between distribution of distinct and non- <br> distinct objects | $\mathrm{PO} 1 \& \mathrm{PO} 2$ | U |
| CO 2 | Interpret the recurrence relation and generating functions and evaluate <br> by using the technique of generating functions | $\mathrm{PO} 3 \& \mathrm{PO} 5$ | A |
| CO 3 | Solve the problems by the principle of inclusion and exclusion | PO 6 | Ap |
| CO 4 | To prove the basic theorems in Boolean Algebra and to develop the <br> truth table for a Boolean expression | PO 4 | Ap |
| CO 5 | Differentiate between variety of lattices and their properties | $\mathrm{PO} 5 \& \mathrm{PO} 7$ | A |

Total Contact hours: 75 (Including lectures, assignments and tests)

| Unit | Module | Topic | Teaching Hours | Cognitive level | Pedagogy | Assessment/ Evaluation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I |  |  |  |  |  |  |
|  | 1. | Permutations and combinations, The rules of sum and product | 3 | K1(R) | Lecture using <br> Chalk and talk,Introductor y session, Group Discussion, Lecture using videos, Problem solving, PPT | Simple definitions, MCQ, Recall formulae |
|  | 2. | Permutations, Combinations | 4 | K2(U) | Lecture using videos, Peer tutoring, Problem solving, Demonstration, PPT, Review | Quiz, MCQ, <br> Recall formulae |
|  | 3. | Distribution of distinct objects | 4 | K4(An) | Lecture using Chalk and talk, Lecture using videos, Problem solving, PPT | Suggest formulae, Solve problems, Home work |
|  | 4. | Distribution of nondistinct objects | 4 | K4(An) | Lecture using Chalk and talk, Lecture using videos, Problem solving, PPT | Class test, Problem solving questions, Home work |
| II |  |  |  |  |  |  |
|  | 1. | Generating functions | 3 | $\begin{aligned} & \text { K1(R), } \\ & \text { K2(U) } \end{aligned}$ | Lecture using Chalk and talk, Problem solving | Simple definitions, MCQ, Recall formulae |
|  | 2. | Generating functions for combinations | 3 | $\begin{aligned} & \text { K3(Ap), } \\ & \text { K2(U) } \end{aligned}$ | Lecture using Chalk and talk, Problem solving | Problem solving, Home work |
|  | 3. | Recurrence relations | 3 | K2(U) | Lecture using Chalk and talk, | Home work |


|  |  |  |  |  | Problem solving |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4. | Linear recurrence relations with constant coefficients | 3 | K3(Ap) | Lecture using Chalk and talk, Group Discussion, Problem solving, PPT | Slip test, Assignments |
|  | 5. | Solution by the technique of generating functions | 3 | K3(Ap) | Lecture using Chalk and talk, Problem solving | Class Test, Problem solving |
| III |  |  |  |  |  |  |
|  | 1. | The principle of  <br> inclusion and <br> exclusion  | 5 | $\begin{gathered} \text { K3(Ap), } \\ \text { K2(U) } \end{gathered}$ | Lecture using Chalk and talk, Group Discussion, Problem solving, PPT | Slip test, Problem solving, Explain |
|  | 2. | The general formula | 5 |  | Lecture using Chalk and talk, Problem solving | Brain Storming, Problem solving |
|  | 3. | Derangements | 5 | K3(Ap) | Lecture using Chalk and talk, Problem solving | Home work |
| IV |  |  |  |  |  |  |
|  | 1. | Boolean Algebra: Introduction | 3 | K2(U) |  | Brain Storming |
|  | 2. | Basic Theorems on Boolean Algebra | 3 | K2(U) | Lecture using Chalk and talk, Problem solving, Peer tutoring | Brain Storming, Problem solving |
|  | 3. | Duality Principle | 3 | K2(U) | Lecture using Chalk and talk, Problem solving, Peer tutoring | Slip test, Assignments |
|  | 4. | Boolean Functions | 3 | K2(U) | Lecture using Chalk and talk, Peer tutoring | Oral test |
|  | 5. | Applications of Boolean algebra | 3 | $\begin{gathered} \mathrm{K} 3(\mathrm{Ap}), \\ \mathrm{K} 4(\mathrm{An}) \end{gathered}$ | Lecture using Chalk and talk, Problem | Brain Storming, Problem solving |

$\left.\begin{array}{|l|l|l|c|c|l|l|}\hline & & & & & \begin{array}{l}\text { solving, Peer } \\ \text { tutoring }\end{array} & \\ \hline \text { V } & \text { 1. } & \begin{array}{l}\text { Posets and Lattices: } \\ \text { Introduction }\end{array} & 2 & \text { K2(U) } & \begin{array}{l}\text { Lecture using } \\ \text { Chalk and talk, } \\ \text { Peer tutoring }\end{array} & \text { Class test } \\ \hline & 2 . & \begin{array}{l}\text { Totally Ordered Set or } \\ \text { Chain }\end{array} & 3 & \text { K2(U) } & \begin{array}{l}\text { Lecture using } \\ \text { Chalk and talk, } \\ \text { Peer tutoring }\end{array} & \begin{array}{l}\text { Problem } \\ \text { solving, } \\ \text { Home work }\end{array} \\ \hline & 3 . & \begin{array}{l}\text { Product Set and Partial } \\ \text { Order Relation }\end{array} & 3 & \text { K3(Ap) } & \begin{array}{l}\text { Lecture using } \\ \text { Chalk and talk, } \\ \text { Peer tutoring }\end{array} & \begin{array}{l}\text { Slip test, } \\ \text { Assignments }\end{array} \\ \hline & 4 . & \begin{array}{l}\text { Hasse Diagrams of } \\ \text { Partially Ordered Sets }\end{array} & 3 & \text { K2(U) } & \begin{array}{l}\text { Graphical } \\ \text { representation, } \\ \text { Demonstration }\end{array} & \begin{array}{l}\text { Suggest } \\ \text { formulae, Solve } \\ \text { problems, } \\ \text { Home work }\end{array} \\ \hline 5 . & \text { Lattice- Duality } & 2 & \text { K4(An) } & \begin{array}{l}\text { Lecture using } \\ \text { Chalk and talk, } \\ \text { Problem } \\ \text { solving, Peer } \\ \text { tutoring }\end{array} & \begin{array}{l}\text { Slip test, } \\ \text { Assignments }\end{array} \\ \hline & 6 . & \text { Types of Lattices } & 2 & \text { K4(An) } & \begin{array}{l}\text { Lecture using } \\ \text { Chalk and talk, }\end{array} & \begin{array}{l}\text { Simple } \\ \text { definitions, } \\ \text { Problem } \\ \text { solving, Peer } \\ \text { tutoring }\end{array}\end{array} \begin{array}{l}\text { MCQ, Recall } \\ \text { formulae }\end{array}\right]$

Course Focussing on Employability/ Entrepreneurship/ Skill Development :Skill Development Activities (Em/En/SD):Evaluation through short test, Seminar

Course Focussing on Cross Cutting Issues (Professional Ethics/ Human Values/Environment Sustainability/ Gender Equity): Nil

Activities related to Cross Cutting Issues: Nil
Assignment : The Tower of Hanoi Problem, Applications of Boolean algebra
Seminar Topic: Basic Theorems of Boolean Algebra and Lattices

## Sample questions

## Part-A

1. Out of a large number of pennies, nickels, dimes and quarters, in how many ways can six coins be selected?
a) 6
b) 84
c) 60
d) 3
2. What is the coefficient of the term $x^{23}$ in $\left(1+x^{5}+x^{9}\right)^{100}$ ?
a) 485500
b) 485000
c) 485100
d) 481000
3. Write the recurrence relation representing the series $1,3,3^{2}, 3^{3}, \ldots, 3^{n}$
4. Say True or False: Boolean algebra has no operation equivalent to subtraction and division.
5. Say True or False: A subset of poset may not have lower or upper bound.

## Part - B

6. If no three diagonals of a convex decagon meet at the same point inside the decagon into how many line segments are the diagonals divided by their intersections.
7. Prove the identity $\binom{n}{0}^{2}+\binom{n}{1}^{2}+\binom{n}{2}^{2}+\cdots\binom{n}{n}^{2}=\binom{2 n}{n}$
8. Solve the recurrence relation using generating function $a_{n}=a_{n-1}+2(n-1)$ with the boundary condition $a_{1}=2$.
9. State and Prove the DeMorgan's Law in Boolean Algebra
10. Prove that the digraph of a partial order has no cycle of length greater than one

## Part - C

11. (i) Find the number of $n$-digit binary sequences that contain an even number of 0 's.?(E)
(ii) What is the number of $n$ digit quaternary sequence that has even number of zero's?
12. (i) Find the number of ways in which 4 persons, each rolling a single die once, can have total score of 17 ?
(ii) What is the ordinary enumerator for the selection of r objects out of n objects ( $r \geq n$ ), with unlimited repetitions, but with each object included in each selection.
13. State and prove the principle of inclusion and exclusion.
14. A committee of three experts for deciding the acceptance or rejection of photographs for exhibition is provided with buggers which members of the committee push to indicate acceptance. Design a circuit so that a bell will ring when there is a majority vote for acceptance.
15. A poset has at most one greatest element and one least element.


Head of the Department
Dr.S.Kavitha


Course Instructor
Ms.Y.A.Shiny

## II PG

## PROGRAMME SPECIFIC OUTCOME (PSOs)

| PSO <br> No. | Upon completion of the M.Sc. Degree Programme, the <br> graduates <br> will be able to: | PO addressed |
| :--- | :--- | :--- |
| PSO - 1 | utilize the knowledge gained for entrepreneurial pursuits. | PO 1 |
| PSO - 2 | sharpen their analytical thinking, logical deductions and rigour in <br> reasoning. | PO 2 |
| PSO - 3 | use the techniques, skills and modern technology necessary to <br> communicate effectively with professional and ethical <br> responsibilities. | PO 3 |
| PSO - 4 | Understand the applications of mathematics ina global economic <br> environmental and societal context. | PO 4 |

## Teaching Plan

| Department | $:$ | Mathematics (SF) |
| :--- | :--- | :--- |
| Class | $:$ | II M.Sc. Mathematics |
| Title of the Course | $:$ | Core IX: Field Theory and Lattices |
| Semester | $:$ | III |
| Course Code | $:$ | PM2031 |


| Course Code | L | T | $\mathbf{P}$ | Credits | Inst. Hours | Total | Mours |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

## Objectives

1. To learn in depth the concepts of Galois Theory, theory of modules and lattices.
2. To pursue research in pure Mathematics.

## Course outcomes

| CO | Upon completion of this course, <br> the students will be able to: | PSO addressed | Cognitive level |
| :---: | :--- | :---: | :---: |
| CO - 1 | recall the definitions and basic <br> concepts of field theory and lattice <br> theory | PSO - 2 | K1 (R) |
| CO - 2 | express the fundamental concepts of <br> field theory, Galois theory | PSO - 2 | K2 (U) |
| CO - 3 | demonstrate the use of Galois theory <br> to construct Galois group over the <br> rationals | PSO - 3 | K5 (E) |
| CO - 4 | distinguish between field theory and <br> Galois theory | PSO - 3 | K3 (A) |
| CO -5 | interpret distributivity and <br> modularity and apply these concepts <br> in Boolean Algebra | PSO - 4 | K3 (A) |
| CO - 6 | understand the theory of Frobenius <br> Theorem | PSO - 2 | K2 (U) |
| CO - 7 | develop the knowledge of lattices <br> and establish new relationships in <br> Boolean Algebra | PSO - 1 | K6 (C) |

Total Contact hours: 90 (Including lectures, assignments and tests)

| Unit | Module | Topics | Teaching Hours | Cognitive level | Pedagogy | Assessment/ Evaluation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I |  |  |  |  |  |  |
|  | 1 | Extension field <br> - Definition, <br> Finite <br> extension- <br> Theorems on <br> finite extension | 4 | K2 (U) | Introductor <br> y session, <br> Lecture <br> with <br> illustration, <br> Problem <br> solving | Questioning, Recall steps, concept definitions, concept with examples |
|  | 2 | Theorems and corollary on algebraic over Fields and understand about subfields of an extension | 4 | K5 (E) | Group <br> Discussion, <br> Lecture <br> with <br> illustration, <br> Problem <br> solving | Evaluation through short test, concept explanations |
|  | 3 | Adjunction of an element to a field, subfields, Theorems. | 4 | K3 (A) | Lecture <br> with <br> illustration, <br> Peer <br> tutoring | Slip Test, concept explanations |
|  | 4 | Algebraic extension, Theorems on algebraic extension, algebraic number, transcendental number | 3 | K3 (A) | Lecture <br> with <br> illustration, PPT | Quiz, concept explanations |
| II |  |  |  |  |  |  |



|  |  | Theorems on simple extension. |  |  |  | through short test |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| III |  |  |  |  |  |  |
|  | 1 | Fixed Field - <br> Definition, <br> Theorems based on Fixed Field, Group of Automorphism | 4 | K2 (U) | Lecture with illustration, Problem solving | concept definitions, concept with examples, Questioning,, Discussion |
|  | 2 | Theorems based on group of Automorphism , Finite Extension, Normal Extension | 5 | K2 (U) | Lecture with illustration, Peer tutoring | concept explanations, Evaluation through short test |
|  | 3 | Theorems based on Normal Extension, Galois Group, Theorems based on Galois Group | 4 | K3 (A) | Lecture with illustration, Group Discussion | concept explanations, Quiz |
|  | 4 | Galois Group over the rationals, Theorems based on Galois Group over the rationals, Problems based on Galois Group | 4 | K3 (A) | Lecture with illustration, PPT | concept explanations |


|  |  |  | over the rationals |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IV |  |  |  |  |  |  |  |
|  | 1 | 1 | Finite Fields - <br> Definition, <br> Lemma-Finite <br> Fields, <br> Corollary- <br> Finite Fields | 4 | K5 (E) | Lecture with illustration, Problem solving | concept with examples, Assignment |
|  | 2 | 2 | Theorems based on Finite Fields, Wedderburn's Theorem on finite division ring | 4 | K3 (A) | Lecture with illustration, Peer tutoring | concept explanations, Quiz |
|  | 3 | 3 | Wedderburn's Theorem, Wedderburn's Theorem-First Proof | 4 | K3 (A) | Lecture with illustration | concept explanations, Evaluation through short test |
|  | 4 | 4 | A Theorem of FrobeniusDefinitions, Algeraic over a field, Lemma based on Algeraic over a field | 3 | K2 (U) | Lecture with illustration, Group Discussion | concept definitions, concept explanations, |
| V |  |  |  |  |  |  |  |
|  | 1 | 1 | Partially ordered setDefinitions, Theorems based on | 3 | K2 (U) | Introductor y session, Lecture with illustration | concept explanations, concept with examples, Seminar |


|  |  | Partially <br> ordered set |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 2 | Totally ordered <br> set, Lattice, <br> Complete <br> Lattice | 4 | K3 (A) | Lecture <br> with <br> illustration | Slip Test, <br> Seminar |
|  |  | Theorems <br> based on <br> Complete <br> lattice, <br> Distributive <br> Lattice | 3 | K2 (U) | Lecture <br> with <br> illustration | concept <br> explanations, <br> Quiz, <br> Seminar |
| 4 | Modular <br> Lattice, <br> Boolean <br> Algebra, <br> Boolean Ring | 4 | K3 (A) | Lecture <br> with <br> illustration | Evaluation <br> through short <br> test, Seminar |  |

Course Focussing on Employability/ Entrepreneurship/ Skill Development: Skill Development
Activities (SD): Find the degree of splitting field of the polynomial.
Course Focussing on Cross Cutting Issues (Professional Ethics/ Human Values/Environment Sustainability/ Gender Equity): Nil

Activities related to Cross Cutting Issues: Nil
Assignment: Finite Fields
Seminar Topic: Lattices and Boolean algebras

## Sample questions

## Part A

1.The degree of an extension field K over a field F is $\qquad$ .
2. If $\mathrm{p}(\mathrm{x}) \in \mathrm{F}[\mathrm{x}]$, then an element $a$ lying in some extension field of F is called a $\qquad$ of $\mathrm{p}(\mathrm{x})$ if $\mathrm{p}(a)=$ 0 .
(b) field
(b) extension field
(c) irrational field
(d) root
3. State True or False: The fixed field of G is a subfield of K.
4. The $\qquad$ group of nonzero elements of a finite field is cyclic.
(a) commutative
(b) cyclic
(c) finite
(d) multiplicative
5. Assertion (A): A lattice homomorphism is order preserving.

Reason (R): Define a homomorphism of a lattice L into a lattice $L^{\prime}$ to be a map $a \rightarrow a^{\prime}$ such that $(a \vee b)^{\prime}=a^{1} \vee b^{\prime}$ and $(a \wedge b)^{\prime}=a^{1} \wedge b^{\prime}$.
(a) Both A and R are true and R is the correct explanation of A .
(b) Both A and R are true and R is not the correct explanation of A .
(c) A is true and R is false.
(d) Both A and R are false.

## Part B

1.Prove that, if L is an algebraic extension of K and if K is an algebraic extension of F then L is an algebraic extension of $F$.
2.State and prove Remainder theorem.
3. If $K$ is a finite extension of $F$, then $G(K, F)$ is a finite group and its order $o(G(K, F))$, satisfies $o(G(K$, F) $) \leq[$ K:F].
4. Let C be the field of complex numbers and suppose that the division ring D is algebraic over C . Then D $=\mathrm{C}$.
5. Show that any totally ordered set is a distributive Lattice.

## Part C

1. If $L$ is a finite extension of $K$ and if $K$ is a finite extension of $F$ then $L$ is a finite extension of $F$. Moreover $[\mathrm{L}: \mathrm{F}]=[\mathrm{L}: \mathrm{K}][\mathrm{K}: \mathrm{F}]$.
2.If $F$ is of characteristic 0 and if $a, b$ are algebraic over $F$, then there exist an element
$C \in F(a, b)$ such that $F(a, b)=F(c)$.
3.State and prove fundamental theorem of Galois theory.
4.Let K be a field and let G be a finite subgroup of the multiplicative group of nonzero elements of K . Then G is a cyclic group.
2. A lattice L is modular if and only if whenever $\mathrm{a} \geq \mathrm{b}$ and $a \wedge c=b \wedge c$ and $a \vee c=b \vee c$ for some c in L , then $\mathrm{a}=\mathrm{b}$.


Head of the Department: Dr.S.Kavitha


Course Instructor: Dr.C.Jenila

## Teaching Plan

Department : Mathematics S.F
Class : II M.Sc Mathematics
Title of the Course : Core X: Topology
Semester : III
Course Code : PM2032

| Course Code | L | T | P | Credits | Inst. Hours | Total <br> Hours | Marks |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PM2032 |  |  |  | CIA | External | Total |  |  |  |
|  | 6 | - | - | 5 | 6 | 90 |  |  |  |
|  |  |  |  |  |  |  | 25 | 75 | 100 |

## Objectives

1. To distinguish spaces by means of simple topological invariants.
2. To lay the foundation for higher studies in Geometry and Algebraic Topology

## Course outcomes

| CO | Upon completion of this course, the students will be able to: | PSO addressed | Cognitive level |
| :---: | :---: | :---: | :---: |
| CO-1 | understand the definitions of topological space, closed sets, limit points, continuity, connectedness, compactness, separation axioms and countability axioms. | PSO-3 | K2(U) |
| CO-2 | construct a topology on a set so as to make it into a topological space | PSO-4 | K3(A) |
| CO-3 | distinguish the various topologies such as product and box topologies and topological spaces such as normal and regular spaces. | PSO-3 | K2(U) |
| CO-4 | compare the concepts of components and path components, connectedness and local connectedness and countability axioms. | PSO-2 | K5(E) |
| CO-5 | apply the various theorems related to regular space, normal space, Hausdorff space, compact space to other branches of mathematics | PSO-1 | K3(A) |
| CO-6 | construct continuous functions, homeomorphisms and projection mappings. | PSO-4 | K6(C) |

Total Contact hours: 90 (Including lectures, assignments and tests)

| Unit | Module | Topic | Teaching Hours | Cogni tive level | Pedagogy | Assessme nt/ Evaluatio n |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I |  |  |  |  |  |  |
|  | 1. | Definition of topology, discrete and indiscrete topology, finite complement topology, Basis for a topology and examples, Comparison of standard and lower limit topologies | 3 | K2(U) | Lecture with Illustration | Evaluatio n through test |
|  | 2. | Order topology: Definition \& Examples, Product topology on XxY: Definition \& Theorem | 3 | K1(R) | Lecture with Illustration | Recall concept definition and steps |
|  | 3. | The Subspace Topology: Definition \& Examples, Theorems | 3 | K3(A <br> p) | Lecture with Examples | Suggest concept with examples, formula |
|  | 4. | Closed sets: Definition \& Examples, Theorems, <br> Limit points: Definition Examples \& Theorems , Hausdorff Spaces: <br> Definition \& Theorems | 5 | K4(A <br> n) | Discussion with Illustration | Differenti ate between various ideas |
|  | 5 | Continuity of a function: Definition, Examples, Theorems, <br> Homeomorphism: Definition \& Examples, Rules for constructing continuous function, Pasting lemma \& Examples, Maps into products | 5 | K2(U) | Lecture with Illustration | Evaluatio n through short test |
| II |  |  |  |  |  |  |
|  | 1 | The Product Topology: Definitions,Comparison | 3 | K6(C) | Lecture with PPT | Check knowledg |


|  |  | of box and product topologies, Theorems related to product topologies, Continuous functions and examples |  |  |  | e in topologies |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | The Metric Topology: <br> Definitions and <br> Examples, Theorems, Continuity of a function, The sequence lemma, Constructing continuous fuctions, Uniform limit theorem, Examples and Theorems | 5 | K3(A) | Lecture with illustration | Solve problems |
|  | 3 | Connected Spaces: Definitions, Examples, Lemmas and Theorems, Connected Sub space of the real lines: Definitions and Examples, Theorems, Intermediate value theorem, connected space open and closed sets, lemma, examples, Theorems. | 5 | K2(U) | Lecture using videos | Evaluatio n through Assessme nt Test |
|  | 4 | Components and Local Connectedness: Definitions, Path components, Locally connected, Locally path connected: Definitions and Theorems | 3 | K3(A) | Group Discussion | Suggest concept with examples, formula |
| III |  |  |  |  |  |  |
|  | 1 | $\begin{array}{lr}\text { Compact } & \text { space: } \\ \text { Definition, } & \text { Examples, }\end{array}$ Lemma, Theorems and Image of a compact space, Product of finitely many compact spaces, Tube lemma, Finite intersection property: Definition \& Theorem | 4 | K2(U) | Lecture with PPT Illustration | Evaluatio n through Quiz |
|  | 2 | Compact Subspaces of the Real Line: Theorem, Characterize compact subspaces of R ${ }^{\mathrm{n}}$, Extreme | 3 | K3(A) | Lecture with Illustration | Suggest formulae with examples |


|  |  | value theorem, The Lebesgue number lemma, Uniform continuity theorem |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3 | Limit Point <br> Compactness:  <br> Definitions, $\quad$ Examples  <br> and Theorems, <br> Sequentially compact  | 2 | K6(C) | Lecture with examples | Check knowledg e through Assignme nt |
|  | 4 | Complete Metric Spaces: Definitions, Examples and Theorems, Isometric embedding | 3 | K5(E) | Group Discussion | Formative Assessme nt Test |
|  | 5 | Compactness in Metric spaces: Totally bounded, Pointwise bounded, Equicontinuous, Definitions, Lemmas, Theorems | 3 | K3(A) | Lecture and group discussion | Evaluatio n through Class test |
| IV |  |  |  |  |  |  |
|  | 1 | Local compactness: Definition \& Examples, Theorems | 3 | K2(U) | Lecture with PPT Illustration | Evaluatio n through MCQ, <br> Short test |
|  | 2 | First Countability axiom, Second Countability axiom: Definitions, Theorems, Dense subset: Definitions \& Theorem, Examples, Lindelof space : Definition, Examples | 3 | K2(U) | Lecture and group discussion | Evaluatio n through Quiz |
|  | 3 | The Separation Axioms: Regular space \& Normal space: Definitions, Lemma, Relation between the separation axioms, Examples based on separation axioms, Theorem based on separation axioms and Metrizable space | 4 | K4(A <br> n) | Lecture with Illustration | Problem solving |
|  | 4 | Normal Spaces: Theorems and Examples | 2 | K2(U) | Lecture with Illustration | Evauation <br> through <br> Test |
|  | 5 | Urysohn lemma | 3 | K3(A) | Lecture | Explain concept |


|  |  |  |  |  |  | with examples |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V |  |  |  |  |  |  |
|  | 1 | Urysohnmetrization theorem, Imbedding theorem | 3 | $\begin{gathered} \mathrm{K} 4(\mathrm{~A} \\ \mathrm{n}) \end{gathered}$ | Lecture with PPT Illustration | Explain concept with examples |
|  | 2 | Tietze extension theorem | 3 | K5(E) | Lecture with Illustration | Evaluatio n through Assessme nt test |
|  | 3 | The Tychonoff Theorem | 3 | K3(A) | Lecture with PPT <br> Illustration | Evaluatio n through Slip test |
|  | 4 | The Stone- <br> CechCompactification:  <br> Defintions, Lemmas, <br> Theorems  | 3 | K3(A) | Lecture with PPT | Explain concept with examples |

Course Focussing on: Skill Development
Activities: Comparison of box and metric topology, group discussion ,seminar
Course Focussing on Cross Cutting Issues (Professional Ethics/ Human Values/Environment Sustainability/ Gender Equity): Nil

Activities related to Cross Cutting Issues: Nil
Assignment : Connected subspace of the Real Line
Seminar Topic: Countable Spaces

## Sample questions

## Part A

2. The standard topology on the real line is. . . . . . .
3. A set $U$ is open in metric topology induced by $d$ iff
(i) For each $y \in U$, there is $\delta>0$ such that $B_{d}(y, \delta) \subset U$
(ii) For each $y \in U$, each $\delta>0$ such that $(y,) \subset U$
(iii) For some $y \in U$, there is $\delta>0$ suchthat $B(y, \delta) \subset U$
(iv) For some $y \in U$, each $\delta>0$ such tha $\mathrm{t}(y, \delta) \subset U$
4. Say true or false

Every compact subspace of a Hausdorff space is closed.
4. When will you say a space is second countable?
5. An arbitrary product of compact spaces is compact in the product topology is the statement of a) Tietze extension theorem b)Tychonoff theorem c) Imbedding theorem.

## Part B

1. Let $A$ be a subset of the topological space $X$. Then prove that $x \in \bar{A}$ if and only if every

Open set $U$ containing intersects $A$.
2. If each space $X_{\alpha}$ is a Hausdorff space, then show that $\Pi$ is a Hausdorff space in Both the box and product topologies.
3. State and prove uniform continuity theorem.
4. Show that every compact Hausdorff space is normal.
5. If $\subset X$ and $f: A \rightarrow Z$ is a continuous map of A into the Hausdorff space Z . Then prove that there is at most one extension of $f$ to a continuous function $g: \bar{A} \rightarrow Z$.

## Part C

6. If $A$ is a subspace of $X$ and $B$ is subspace of $Y$, then prove that the product topology on $A \times B$ is same as the topology $A \times B$ inherits as a subspace of $X \times Y$.
7. State and prove the intermediate value theorem.
8. Let $X$ be a metrizable space. Then prove that the following are equivalent:
(i) $\quad X$ is compact.
(ii) $X$ is limit point compact.
( iii) $X$ is sequentially compact
9. State and prove Urysohn Lemma.
10. State and prove Tietze extension theorem.


Head of the Department:
Dr. S. Kavitha

## Dr.J.Nesa Golden Flower

## Teaching Plan

| Department | : Mathematics (S.F) |
| :--- | :--- |
| Class | : II M.Sc Mathematics |
| Title of the course | : Measure Theory and Integration |
| Semester | $:$ III |
| Course Code | $:$ PM2033 |


| Course Code | L | T | P | Credits | Inst. Hours | Total Hours | Marks |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | CIA | External | Total |
| MC2053 | 5 | - | - | 5 | 5 | 90 | 25 | 75 | 100 |

## Objectives:

1.To generalize the concept of integration using measures.
2. To develop the concept of analysis in abstract situations
3. To apply Measures and Lebesgue integrals to various Measurable sets and Measurable functions
4.To compare the different types of measures and signed measure

## Course Outcomes:

| CO | Upon completion of this course the students willbe <br> able to : | PSO addressed | Cognitive <br> level |
| :---: | :--- | :---: | :---: |
| CO-1 | Define the concept of measures and Vitali covering and recall <br> some properties of convergence of functions, | $\mathrm{PSO}-1$ | U |
| CO-2 | Cite examples of measurable sets, Measurable functions, <br> Riemann integrals, Lebesgue integrals. | $\mathrm{PSO}-3$ | A |
| CO-3 | Apply measures and Lebesgue integrals to various measurable <br> sets and Measurable functions | $\mathrm{PSO}-2$ | Ap |
| CO-4 | Apply outer measure, Differentiation and integration to <br> intervals, functions and Sets. | $\mathrm{PSO}-2$ | Ap |
| CO-5 | Compare the different types of Measures and Signed measure <br> Construct Lp Spaces and Outer Measurable Sets. | $\mathrm{PSO}-3$ | A |

## Total Contact hours : 90 (Including lectures, assignments and tests)



|  | 4. | Differentiation of an <br> integral | 4 | K2(U) | Lecture and Group <br> discussion | Test |
| :---: | :---: | :--- | :--- | :---: | :--- | :--- |
|  | 5. | Absolute continuity. | 3 | K1(R) | Lecture using Chalk <br> and talk | Evaluation |

## IV Measure and integration

| 1. | Introduction of Measure <br> and integration | 4 | K2(U) | Lecture using Chalk <br> and talk | Recall definitions and <br> basic concepts |  |
| :---: | :---: | :--- | :--- | :---: | :--- | :--- |
|  | 2. | Measure spaces | 2 | K1(R) | PPT | Assignment |
| 3. | Measurable functions | 2 | K2(U) | Lecture with <br> Illustration | Recall different types <br> of functions |  |
| 4. | Integration | 3 | K3(Ap) | Lecture with <br> Illustration | Concept explanation <br> with examples |  |
| 5. | General Convergence <br> Theorems | 2 | K1(R) | Group discussion | Evaluation through <br> short test, concept <br> explanations |  |
|  | 6. | Signed measures. | 2 | K2(U) | Lecture using Chalk <br> and talk | Evaluation |
| V The Lp spaces |  |  |  |  |  |  |


| 1. | Introduction of The LP <br> spaces | 5 | K2(U) | Group discussion | Q\& A |  |
| :---: | :---: | :--- | :--- | :--- | :--- | :--- |
|  | 2. | Measure and outer <br> measure | 4 | K1(R) | Group discussion | Assignment |
| 3. | Outer measure and <br> measurability | 3 | K3(Ap) | Lecture with <br> Illustration | Test |  |
| 4. | The extension theorem | 3 | K3(A) | Lecture with PPT | Evaluation |  |

Course Focussing on Employability/Entrepreneurship/Skill Development : Employability
Activities(Em/En/SD) : Evaluation through short test,Seminar.
Course Focussing on Cross Cutting Issues (Professional Ethics/ Human Values/Environment Sustainability/ Gender Equity): Nil

Activities related to Cross Cutting Issues: Nil

Seminar Topic: Measure and Outer Measure

## Sample questions

## Part A

1. If $\mathrm{A}, \mathrm{B} \in \mu$ and AcB then $\qquad$
2.Outer Measure of a Countable set is $\qquad$
2. A linear combination of the set E is said to be $\qquad$
4.If f is absolute continuity on $[\mathrm{a}, \mathrm{b}]$ then it is of $\ldots \ldots \ldots \ldots .$. on $[\mathrm{a}, \mathrm{b}]$
5.Every $\sigma$ - finite measure is. $\qquad$

## Part B

1.State and Prove monotonicity property
2.State and Prove Fatou's lemma
3. Show that function f is bounded variation on $[\mathrm{a}, \mathrm{b}]$ iff f is the difference of two monotone real valued function on $[\mathrm{a}, \mathrm{b}]$
4.Let $f$ be an increasing real valued function $[a, b]$. Then $f$ is differential a.e.prove that the $f^{\prime}$ is measurable
5.If $E_{i} \in \beta, \mu E_{1}<\infty$ and $E_{i} \supset E_{i+1}$ then $\mu\left(E_{i}\right)=\mu E_{n}$

## Part C

1. Prove that the outer measure of an interval is its length.
2.Prove that every boral set is measurable. In particular each open set and each closed set is measurable.
3.State and Prove Bounded Convergence Theorem
2. Show that if f is absolute continuity on $[\mathrm{a}, \mathrm{b}]$ then it is of bounded variation on $[\mathrm{a}, \mathrm{b}]$
3. State and Prove The extension theorem


## Head of the Department Dr.S.Kavitha



Course Instructor Ms.Y.A.Shiny

## Teaching Plan

| Department | $: \quad$ Mathematics S.F. |
| :--- | :--- |
| Class | $:$ |
| II M.Sc Mathematics |  |

Title of the Course : Elective III: Algebraic Number
Theory and Cryptography
Semester : III
Course Code : PM2034

| Course Code | $\mathbf{L}$ | T | $\mathbf{P}$ | Credits | Inst. Hours | Total | Marks |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Hours | CIA | External | Total |  |
| PM2034 | $\mathbf{4}$ | $\mathbf{2}$ | - | $\mathbf{4}$ | 6 | $\mathbf{9 0}$ | $\mathbf{2 5}$ | $\mathbf{7 5}$ | $\mathbf{1 0 0}$ |

## Objectives

- To gain deep knowledge about Number theory
- To study the relation between Number theory and Abstract Algebra.
- To know the concepts of Cryptography.


## Course outcomes

| CO | Upon completion of this course, <br> the students will be able to: | PSO addressed | Cognitive level |
| :---: | :--- | :---: | :---: |
| CO - 1 | Recall the basic results of field <br> theory | PSO -1 | K1(R) |
| CO -2 | Understand quadratic and power <br> series forms and Jacobi <br> symbol | PSO -2 | K2(U) |
| CO - 3 | Apply binary quadratic forms for <br> the decomposition of a <br> number into sum of sequences | PSO -3 | K3(AP) |
| CO -4 | Determine solutions using <br> Arithmetic Functions | PSO -3 | K3AP ) |
| CO -5 | Calculate the possible partitions of a <br> given number and draw <br> Ferrer's graph | PSO -2 | K4(An ) |
| CO-6 | Identify the public key using <br> Cryptography | PSO -4 | K4(An ) |

Total Contact hours: 90 (Including lectures, assignments and tests)

| Unit | Module | Topic | Teaching Hours | Cognitive level | Pedagogy | Assessment/ Evaluation |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I |  |  |  |  |  |  |
|  | 1. | Quadratic ResiduesDefinition and examples, Legendre symbol definition and Theorem | 2 | K2(U) | Lecture using Chalk and talk, Group Discussion, Problem solving, Lecture using PPT | Recall basic definitions on Number theory, Solving problems |
|  | 2. | Lemma of Gauss, Theorem on Legendre symbol | 3 | K1(R) | Lecture using PPT, <br> Problem solving | Evaluation through short test, concept explanations, Solving problems |
|  | 3. | Quadratic reciprocity- The Gaussian reciprocity law, Theorem on Quadratic reciprocity | 3 | K3(Ap) | Lecture using Chalk and talk, Lecture using PPT, Problem solving. | MCQ, Solving problems, <br> Questioning and Home Assignment |
|  | 4. | The Jacobi symbolDefinition, examples and Theorems | 3 | K4(An) | Lecture using Chalk and talk, Lecture using PPT, Problem solving | Evaluation through short test, Seminar, MCQ and Home Assignment |
| II | Binary Quadratic forms |  |  |  |  |  |
|  | 1. | Binary Quadratic <br> forms- <br> examples <br> Definition,  <br> Theorems and | 3 | K2(U) | Lecture using PPT, Group Discussion, Problem solving | Concept explanations with examples, True / False |
|  | 2. | Equivalence and Reduction of Binary Quadratic formsDefinition and Theorems | 4 | K2(U) | Lecture using PPT, Group Discussion, Problem solving | Class test, Simple definitions ,examples, MCQ and Home Assignment |


|  | 3. | Sum of two squaresTheorems on sum of two squares | 4 | K3(AP) | Lecture using Chalk and talk, Lecture using PPT, Problem solving | MCQ, Recall <br> steps, <br> Questioning and <br> Home <br> Assignment |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| III | Some Functions of Number Theory |  |  |  |  |  |
|  | 1. | Arithmetic functions- Definition, examples and Theorems | 3 | K3(AP ) | Lecture using Chalk and talk, Lecture using PPT, Problem solving | Evaluation through short test, concept explanations |
|  | 2. | The Mobius Inversion Formula- Definition, Theorems and Problems | 3 | K3(AP ) | Lecture using Chalk and talk, Lecture using PPT, Problem solving | Peer teaching, MCQ, Recall steps, <br> Questioning and Home Assignment |
|  | 3. | Some Diophantine Equations - Examples and Theorems | 3 | K4(An ) | Lecture using Chalk and talk, Lecture using PPT, Problem solving | Simple definitions ,examples, Evaluation through asking question |
|  | 4. | Pythagorean Triangle- Definition, Lemma and Theorems | 3 | K4(An ) | Lecture using Chalk and talk, Lecture using PPT, Problem solving | MCQ, Recall <br> steps, <br> Questioning and <br> Home <br> Assignment |
| IV | The partition Function |  |  |  |  |  |
|  | 1. | Partitions definitionsDefinition, examples and Theorems | 3 | K4(An ) | Lecture with Illustration, Lecture using PPT, Problem solving | Peer teaching, MCQ, Recall steps, Questioning and Home Assignment |
|  | 2. | Ferrers $\quad$ Graphs- Definition, examples and Theorems | 2 | K4(An ) | Lecture with Illustration, Lecture using PPT, Problem solving | Questioning and Home Assignment |
|  | 3. | Formal power series, Generating Functions and Euler's identityDefinition, examples and Theorems | 3 | K4(An ) | Lecture with Illustration, Lecture using PPT, Problem solving | Class test, Questioning and Home Assignment |


|  | 4. | Euler's identity and bounds on $\mathrm{p}(\mathrm{n})$ Lemma and Theorems | 4 | $\begin{gathered} \text { K4(An), } \\ \text { K5(E) } \end{gathered}$ | Lecture with Illustration, Lecture using PPT, Problem solving | Peer teaching, MCQ, Recall steps, <br> Questioning and Home Assignment |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| V | Public Key Cryptography |  |  |  |  |  |
|  | 1. | The concepts of <br> Public Key <br> Cryptography - <br> Definition and <br> examples  | 3 | K4(An ) | Lecture with Illustration, Lecture using PPT, Problem solving, Peer teaching. | Peer teaching, MCQ, Recall steps, Seminar Questioning and Home Assignment |
|  | 2. | RSA Cryptosystem with examples | 2 | K4(An ) | Lecture with Illustration, Lecture using Videos, Problem solving, Peer teaching | Questioning ,Seminar |
|  | 3. | Discreter log <br> cryptosystem with | 3 | K4(An ) | Lecture with Illustration, Lecture using PPT, Problem solving, Peer teaching | Suggest idea/concept examples, Seminar |
|  | 4. | The Massy- Omura cryptosystem for message transmission, the ElGamal cryptosystem, the Digital Signature Standard, Algorithm for finding discrete log in finite fields with example and index calculus algorithm for discrete logs | 4 | K4(An ) | Lecture with Illustration, Lecture using PPT, Problem solving, Peer teaching | Evaluation through asking question, Home Assignment, Seminar |
|  | 5. | Basic facts of Elliptic curves, Elliptic curves over the reals, complexes and rationales, Points of finite order with examples | 3 | K4(An ) | Lecture with Illustration, Lecture using PPT, Problem solving, Peer teaching | Evaluation through short test, Seminar |


| 6. | Analog of the Diffie- <br> Helman key exchange, <br> Analog of Massey - <br> Omura, Analog of <br> ElGamal, reducing a <br> global modulo p with <br> examples | K4(An ) | Lecture with <br> Illustration, | Recall steps, <br> Seminar |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Lecture using |  |  |  |  |
| videos, |  |  |  |  |
| Problem |  |  |  |  |
| solving, Peer |  |  |  |  |
| teaching |  |  |  |  |\(\quad\left\{\begin{array}{l} <br>

\hline\end{array}\right.\)

Course Focussing on : Skill Development
Activities: Solving problems in Legendre symbol, Solving problems in different type of forms, Seminar, Online Quiz, Pythagorean triangles through videos.

Course Focussing on Cross Cutting Issues (Professional Ethics/ Human Values/Environment Sustainability/ Gender Equity): Nil

Activities related to Cross Cutting Issues: Nil
Assignment : Problems on Quadratic Residues, Problems on Binary Quadratic forms Problems on Pythagorean Triangle and Elliptic curves

Seminar Topic: The partition Function, Public Key Cryptography

## Sample questions

## Part A

1. The value of $\left(\frac{7}{13}\right)=$ $\qquad$
2. The form $\mathrm{f}(x, y)=x^{2}+y^{2}$ is called $\qquad$
a) Definite
b) indefinite
c) positive definite
d) positive semi definite
3. The value of ' $\Omega(12)$ is $\qquad$
a) 1
b) 3
c) 5
d) None of the above
4. The conjugate of the conjugate partition is the $\qquad$
5. Public-key cryptography is also known as $\qquad$
a) Asymmetric cryptography
b) Symmetric cryptography
c) Both A and B
d) None of the above

## Part B

1. State and prove Gaussian Reciprocity law.
2. Let $f(x, y)=a x^{2}+b x y+c y^{2}$ be a binary quadratic form with integral coefficients and discriminant $d$. If $d \neq 0$ and $d$ is not a perfect square, then prove that $a \neq 0, c \neq 0$ and the only solution of the equation $f(x, y)=0$ in integers is $x=y=0$.
3. For every positive integer $\mathrm{n}, \sigma(\mathrm{n})=\prod_{p^{\alpha} / / n}\left(\frac{p^{\alpha+1}-1}{p-1}\right)$
4. Derive Euler's identity.
5. Write a short note on authentication.

## Part C

1. If p is an odd prime and $(\mathrm{a}, 2 \mathrm{p})=1$, then $\left(\frac{a}{p}\right)=(-1)^{t}$ where $\mathrm{t}=\sum_{j=1}^{\frac{(p-1)}{2}}\left[\frac{j a}{p}\right]$.

Also $\left(\frac{2}{p}\right)=(-1)^{\frac{\left(p^{2}-1\right)}{8}}$.
2. Let n and d be given integers with $\mathrm{n} \neq 0$. Prove that there exists a binary quadratic form of discriminant $d$ that represents $n$ properly iff the congruence $x^{2} \equiv d(\bmod 4|n|)$ has a solution.
3. State and prove Mobius inversion formula.
4. If $\mathrm{n} \geq 0$, prove that $q^{e}(n)-q^{o}(n)=\left\{\begin{array}{c}(-1)^{j} \text { if } n=\left(3 j^{2} \pm j\right) / 2 \text { for some } j=0,1,2, \ldots \\ 0 \quad \text { otherwise }\end{array}\right.$
5. Explain the Diffle-Hellman key exchange system.


Head of the Department
Dr.S.Kavitha


## Course Instructor

Dr.S.Kavitha

